Mersin et al., *Cogent Engineering* (2017), 4: 1415107
https://doi.org/10.1080/23311916.2017.1415107

PRODUCTION & MANUFACTURING | RESEARCH ARTICLE

A new method for calculating fuel consumption and displacement of a ship in maritime transport

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Received: 30 September 2017
Accepted: 06 December 2017
First Published: 11 December 2017

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Reviewing editor:
Wei Meng, Wuhan University of Technology, China

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Abstract: Fuel consumption is the most important parameter which is effected by the petrol prices. Optimization of the sailing speed decreases the fuel cost because it is proportional to the sailing speed. In this study, the present fuel consumption formula has been improved by finding a formula that shows the change of the displacement of the ship according to the time.

Subjects: Engineering Mathematics; Logistics; Operations Management; Energy & Fuels

Keywords: displacement; optimum speed; fuel consumption

1. Introduction

The total cost usually accounts for about 12–25% although the fuel and oil costs are much higher for some ship types. Especially, when the fuel prices are high, fuel cost is the most important part of the voyage costs (Chrzanowski, 1989).

Although technology is improving day by day, fuel costs still remain important. Fuel costs vary depending on the type of ship's machine, the horsepower of the machine, the type of fuel used and the unit price of the fuel. The regular maintenance of the main machine, the training and the experience of the personnel working in the machine room are other important factors affecting the fuel consumption (Yıldız, 2008).

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PUBLIC INTEREST STATEMENT

Fuel consumption is one of the problem in maritime transportation. There are some equations and formulas for calculating consumption. Some of them neglects the weight of the ship while calculating. However, Barras' formula does not neglect the weight of the ship and according to Barras' consumption formula, fuel consumption can be calculated without time parameter. There is a method, which uses the formula to calculate consumption, is called "calassical method" in this study. But, this calculation is not enough to make accurate estimation. In this study, we found an alternative method to calculate fuel consumption which is called "new method" and we compared these two methods.

The part of the ship below the sea is resistant to the movement of the ship. This resistance affects the fuel consumption of the main machine depending on the speed of the ship. In addition, some variables that can not be controlled outside of them also affect fuel consumption. For example, depending on weather and marine conditions, the malin engine is forced or the fuel combustion conditions change, which also affects fuel consumption (Yıldız, 2008).

However, the most important factor affecting fuel consumption is the service speed of the ship. In general, high service speed has both advantages and disadvantages. The first advantage is the amount of cargo transported annually. For example, the ship which has 7 leged route as Algeciras-Barcelona-Valencia-Marseille-Genoa-Gioia Tauro—İstanbul, has to sail 4,994 miles. It takes about 13 days to sail this distance with a speed of 15 knots for a 10,000 ton capacity gateways. If we accept that the loading and unloading operations at the ports last for 1 day, the moving time of 10,000 tons of cargo will be 24 days in total. This means approximately 152,083.33 tons of cargo per year. If the ship had transported at 20 knots, the service time would be 21 days and this means approximately 173,809.52 tons of cargo per year. The second advantage is inventory cost. The cargos which is carried by cantainers have high inventory costs. For example, Notteboom (Notteboom & Cariou, 2013) estimates that 1 day delay of a 4000 TEU cantainer ship causes €57,000 cost.

2. Literature review

Alderton (1981) published a formula for consumption of a ship.In this formula, weight of the ship was neglected. Then Ronen (1982) and Chrzanowski (1989) used this formula in their studies. Barras (2004) publised a formula for fuel consumption which does not neglect the weight of the ship. Christiansen, Fagerholt, Nygreen, and Ronen (2013) compiled to 131 articles which are published journals about subject of ship routing and scheduling between 2002 and 2012. They presented methods which used in their study. Kim, Chang, Kim, and Kim (2012) determined amount of fuel and optimum vessel speed for a specific vessel route. The study was solved the problem by using epsilon-optimal algorithm. Notteboom and Cariou (2013) researched effects of slow speed applications. Also they analyzed fuel consumption and BAF which paid by shippers. Khor, Døhlie, Konovessis, and Xiao (2013) set up a soft ware to optimize speed of ultra container vessels. They found out optimum speed as 19.5 knot. Sheng, Lee, and Chew (2014) investigated to how determined vessel speed as dynamical from current seaport to next seaport (Psaraftis & Kontovas, 2013). Fagerholt, Laporte, and Norstad (2010) solved the optimal speed on a specific route problem (Drewry, 2014). Doudnikoff and Lacoste (2014) presented differences speed and cost-effectiveness between inside and outside SECA amount of CO_2 emissions of the total transit time.

3. New formula for fuel consumption

Empiric datas show that fuel consumption varies geometrically with increasing speed. For example, at some speeds, when you increase your ship's speed by 30%, fuel consumption increases by twice the initial speed. However, daily consumption varies by up to 6%, depending on whether the ship is full loaded or on ballast. While the ships are anchored in the port, they consume approximately 15% of the fuel consumption at sea (Chrzanowski, 1989).

In the case of ignoring the load on the ship, there is a classical relation between speed and fuel consumption (Wang & Meng, 2012). That is

$$F(v) = \lambda \cdot v^{\Omega} \tag{1}$$

$\lambda > 0; \Omega > 1$

In this relation, F represents fuel cosumption function. Ω can not be equal 1 otherwise fuel consumption would be linear but we know that fuel consumption graph is a curve. However, Ω must be greater than 1. Otherwise consumption function would be decreasing function by speed. λ parameter is a constant coefficient that each ship motor has. Ronen (1982) stated that the approximate rate of daily fuel consumption is proportional to the cube. Wang and Meng (2012) determined that Ω is between 2.7 and 3.3 for container ships with the help of historical data and verified Ronen's statement. Similarly, they showed that Ω is equal 3.5 for the feeder container ships, Ω is equal 4 for medium sized container ships and Ω is equal 4.5 for very large sizes. However, Barras showed that fuel consupmtion formula is (Barras, 2004)

$$F(v) = \lambda v^3 \nabla^{\frac{2}{3}} \tag{2}$$

where ∇ is displacement of the ship. In this study, this formula is called *Classical Formula*.
Load and time factors have been neglected in the classic fuel consumption formula. One of the important factors in the maritime transport is to be at the port in time. However, depending on the weather conditions along the way, arrival times at ports can be flexible. Therefore, the time factor should be added when calculating the length of the route and the fuel consumption. In this study, we noticed that displacement according to time was ignored in classical formulas and we have formulated the present displacement in the following manner in a change of Δt from the starting moment.

THEOREM 2.1 *If we denote the initial displacement of the ship, with $\nabla(0)$, displacement of a ship at a time t is*

$$\nabla(t) = \left[\sqrt[3]{\nabla(0)} - \frac{\lambda v^3 t}{3}\right]^3 \tag{3}$$

Proof The displacement of the ship at any time is the sum of weight of the load on the ship and the weight in tonnes of fuel in the tank. We know that $F(v) = \lambda v^3 \cdot \nabla^{\frac{2}{3}}$ is the formula of the fuel consumption in a day. So, $\Delta t \cdot F(c)$ tonnes of fuel will be consumed Δt moment later. If we denote the displacement of the ship at a time t by $\nabla(t)$, the displacement will be as below Δt moment later

$$\nabla(t + \Delta t) = \nabla(t) - t \cdot F(v(t)) = \nabla(t) - t \cdot \lambda v^3 [\nabla(t)]^{\frac{2}{3}}$$

Here, $v(t)$ is the speed of the ship at t.

$$\frac{\nabla(t + \Delta t) - \nabla(t)}{\Delta t} = -\lambda v^3 [\nabla(t)]^{\frac{2}{3}}$$

For $\Delta t \to 0$

$$\lim_{t \to 0} \frac{\nabla(t + \Delta t) - \nabla(t)}{\Delta t} = \nabla'(t) = -\lambda v^3 [\nabla(t)]^{\frac{2}{3}}$$

So, we have a differential equation $y' = A \cdot y^{\frac{2}{3}}$ by arranging the equation.

$$\frac{\nabla'(t)}{[\nabla(t)]^{\frac{2}{3}}} = -\lambda v^3 \Rightarrow \int \frac{\nabla'(t)}{[\nabla(t)]^{\frac{2}{3}}} dt = \int -\lambda v^3 dt$$

$$\nabla(t) = u \Rightarrow \nabla'(t)\mathrm{d}t = \mathrm{d}u$$

by variable transformation

$$\int \frac{du}{u^{\frac{2}{3}}} = -\lambda v^3 t + c$$

$$\Rightarrow 3\sqrt[3]{\nabla(t)} = -\lambda v^3 t + c$$

$$\Rightarrow \nabla(t) = \left[\frac{c}{3} - \frac{\lambda v^3 t}{3}\right]^3$$

$$\Rightarrow \nabla(0) = \left[\frac{c}{3}\right]^3 \Rightarrow 3 \cdot \sqrt[3]{\nabla(0)} = c$$

$$\Rightarrow \nabla(t) = \left[\sqrt[3]{\nabla(0)} - \frac{\lambda v^3 t}{3}\right]^3$$

Therefore, the desired result is obtained.

In the above suggestions, the elapsed time is taken as day but, if we need to calculate the time by hour, the equaton will be

$$\nabla(t) = \left[\sqrt[3]{\nabla(0)} - \frac{\lambda v^3 t}{72}\right]^3 \tag{4}$$

If we denote the amount of fuel remaining at a time t with $L(t)$

$$L(t) = \nabla(t) - W_{i,i+1} \tag{5}$$

$W_{i,i+1}$ $(1 \leq i \leq n)$ is the weight of the cargo which is carried from port i to port $(i + 1)$ by the ship. The amount of fuel the ship consumes along the route is

$$L(0) - L(t_{n-1}) \tag{6}$$

t_i $(1 \leq i \leq n)$ is the time which passes between port i and port $(i + 1)$.

Example 2.1.1 The route of a vessel with an initial fuel weight of 229,000 tons and a total displacement of 235,000 tons is given by the following table. The table shows the amount of containers on board at the exit from the ports and the time of travel between the two ports

$A \rightarrow B$	10 h	6,000 ton
$B \rightarrow C$	15 h	5,500 ton
$C \rightarrow D$	20 h	6,500 ton

According to the table, there is 6,000 ton weighted cargo on board at A–B leg and the ship time is 10 h. Likewise, there is 5,500 ton weighted cargo on board at B–C. It means 500 ton cargo handeled out at port B and the ship time is 15 h at this leg. Also, there is 6,500 ton weighted cargo on board at B–C. It means 1,000 ton cargo handeled in at port C and the ship time is 20 h at this leg.

If the design speed of the ship is 15 kt and $\lambda = \frac{1}{110.000}$ calculate the total amount of fuel consumed with both the classical method and the new method.

Classical Method:

First, we calculate the leg A–B. The weight of the ship is 235,000 ton and the design speed is 15 kt. The classical method neglects the changing of fuel weight. So the consumption at A–B leg is

$$F_{A\rightarrow B}(v(10)) = \frac{10}{24} \cdot \frac{1}{110,000} \cdot 15^3 \cdot (235,000)^{\frac{2}{3}} = 48.683496 \text{ ton.} \tag{7}$$

Now we calculate the leg B-C. The weight of the ship is

235,000 − 48.68349609 = 234,951.3165 ton and the consumption at B–C leg is

$$F_{B\rightarrow C}(v(15)) = \frac{15}{24} \cdot \frac{1}{110,000} \cdot 15^3 \cdot (234,951.3165)^{\frac{2}{3}} = 73.01515832 \tag{8}$$

So, the last weight of the ship is 234,951.3165 – 73.01515832 = 234,878.3013 ton. The consumption at the last leg is

$$F_{C\to D}\big(v(20)\big) = \frac{20}{24}\cdot\frac{1}{110,000}\cdot 15^3\cdot(234,878.3013)^{\frac{2}{3}} = 97.33337385 \tag{9}$$

Total consumption: 219.0320283 ton.

New Method:

First, we calculate the amount of fuel that will be remain at the end of 10 h when we go on the road with the initial displacement. To do this we first need to find $\nabla(10)$.

$$\nabla(10) = \left[\sqrt[3]{235,000} - \frac{1}{110,000}\cdot\frac{15^3\times 10}{72}\right]^3 = 234,951.3199 \tag{10}$$

This equality gives the total weight of the load and the fuel in the tank at the end of 10 h.

However, cargo on board is 6,000 ton at the $A\to B$ leg so the fuel in the tank at the end of 10 h is

$$L(10) = 234,951.3199 - 6,000 = 228,951.3199 \tag{11}$$

Now we calculate the $B\to C$ leg. Displacement of the ship is 234,451.3199 ton when the time it leaves port B.

Displacement at the end of the 15th hour from exit B is

$$\nabla(15) = \left[\sqrt[3]{234,451.3199} - \frac{1}{110,000}\cdot\frac{15^3\times 15}{72}\right]^3 = 234,378.4159 \tag{12}$$

$$\begin{gathered}\text{At } B\to C \text{ leg, the load on the ship is } 5,500 \text{ tons, and at the end of } 15\\ \text{h the amount of fuel in the tank will be } L(15) = 234,378.4159 - 5,500 = 228,878.4159\end{gathered} \tag{13}$$

Now we calculate the $C\to D$ leg. Displacement of the ship is 235,378.4159 ton when the time it leaves port C.

Displacement at the end of the 20th hour from exit C is

$$\nabla(20) = \left[\sqrt[3]{235,378.4159} - \frac{1}{110,000}\cdot\frac{15^3\times 20}{72}\right]^3 = 235,280.9579 \tag{14}$$

$$\begin{gathered}\text{At } C\to D \text{ leg, the load on the ship is } 6,500 \text{ tons, and at the end of } 20\\ \text{h the amount of fuel in the tank will be } L(20) = 235,280.9579 - 6500 = 228,780.9579\end{gathered} \tag{15}$$

Finally, total fuel consumption: 229,000 – 228,780.9579 = 219.0421

As we can see in the above example, the fact that both of the results are almost the same shows that the work we are doing is correct, but the fact that we can find the displacement at any given time seems to be the most important difference that distinguish us from the classical method. Moreover, this method gives the closest result to the consumption of the fuel until the time *t*.

5. Conclusions

In this study, we found the theorem $\nabla(t) = \left[\sqrt[3]{\nabla(0)} - \frac{\lambda v^3 t}{3}\right]^3$ which allows us to find out what the displacement of the ship is at any given time by introducing the time parameter which is the deficiency of the classical formula of $F(v) = \lambda \cdot v_{ij}^{\Omega} \nabla^{\frac{2}{3}}$ and we can calculate the fuel consumption at any given time t with the equation $L(0) - L(t)$. Classical method and new method were compared by applying on Example 2.1.1. As a result of this comparison, the fuel consumption obtained by the classical method is 219.0320283 tons while the fuel consumption by the new method is 219.0421 tons. The close proximity of these two results indicates that the calculation is correct, but it is a more accurate calculation.

In addition, the new formula can be applied on a ship which does not have a constant speed. Of course, shipping companies define an *eco speed* for their ships and the ship stays this speed along the voyage. However, the speed of the ship can not be fixed due to various reasons (weather opposition etc). So, if a ship has a speed which changes by time, the new method can calculate the fuel consumption for any given time despite classical method will be failed calculating the consumption.

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: A new method for calculating fuel consumption and displacement of a ship in maritime transport, Kadir Mersin, Güler Alkan & Tunç Mısırlıoğlu, *Cogent Engineering* (2017), 4: 1415107.

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